

ESTIMATING THE DEMAND UNCERTAINTY IN SINGLE-
PERIOD INVENTORY PROBLEMS USING THE GOMPERTZ
CURVE AND THE SCHMEISER-DEUTSCH DISTRIBUTION

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THESIS

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USING THE GOMPERTZ CURVE AND THE
SCHMEISER-DEUTSCH DISTRIBUTION

by

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Estimating the Demand Uncertainty
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the Gompertz Curve and the Schmeiser-Deutsch Distribution

by

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ABSTRACT

The optimal order quantity for the case of uncertainty in the newsboy problem depends on the maximum demand. In previous work it was assumed that the demand distribution was known. In practice, this is often not the case. This study suggests some procedures which can be used to estimate the demand distribution even if data on unsatisfied demands are not available.

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I. INTRODUCTION

The perishable product is beginning to receive attention in the inventory problem. Often, it has a fixed lifetime after which it becomes useless to satisfy demands. Newspapers, medical drugs, human blood, photographic films and some foodstuffs are common examples.

An inventory problem for perishable products is often treated as a single-period inventory problem, sometimes called a newsboy problem. The newsboy problem was introduced by Morse and Kimball in 1950 and the development of it has been presented in several journals and publications. A brief history of it was given by Masuda [Ref. 1].

A well-known characterization of the newsboy problem is:

The newsboy must decide how many papers to purchase for resale during the day. If he buys too many papers--that is, more papers than people will purchase from him--he will have papers left at the end of the day and will incur a loss on each, since old newspapers have little salvage value. If, on the other hand, he purchases too few papers for resale, he will sell out early and his later customers will not be able to buy from him; he thus incurs a cost (lost profit) for each paper he could have sold, but didn't. The newsboy's problem is to decide how many papers to have on hand at the beginning of the day so as to minimize the losses (due to surplus or shortage) at the end of the day. [Ref. 2]

Finding good solutions to the newsboy problem depends upon the amount of information about the magnitude of demand X.

In recent research dealing with newsboy ordering decisions where little information existed about demand, it was

assumed that at the end of each period the demand for that period would be known [Refs. 1,3,4]. In order for this to happen, one would need unlimited supply or some way to measure the demand from unsatisfied customers. In practice, this is often not the case. For example, when the newsstand is empty, the later customers will not stop at that newsstand. Accordingly, we do not know how many more customers would like to buy that newspaper. Thus, we may have frequency data on demand only up to the value of supply.

This thesis is intended to provide methods permitting solutions to the newsboy problem when we only have demand frequencies up to the supply value.

In Chapter II, two procedures will be proposed to estimate the maximum demand. The first procedure uses the Gompertz Curve since the nature of the demand cumulative distribution function (c.d.f.) can be viewed as a growth curve with the market saturate as an asymptote. The second procedure suggested is the four-parameter Schmeiser-Deutsch Distribution since it can take on a wide variety of shapes so as to match many distributions. Also, the Schmeiser-Deutsch Distribution was found to be a suitable procedure for the case of limited supply.

Using the estimation of maximum demand together with the unit cost of surplus and the unit cost of outage, the estimation of optimal order quantity is discussed in Chapter III for the "under uncertainty" and "under risk" cases.

Conclusions and applications are given in Chapter IV.
A review of the Schmeiser-Deutsch Distribution is presented
in Appendix A.

II. ESTIMATION OF MAXIMUM DEMAND

Care should be taken to estimate the maximum demand since the optimal ordering quantity for uncertainty depends on it. In this chapter, two alternatives are suggested to estimate the maximum demand when we have unlimited supply. Then, we will show how one of these methods may be used to estimate maximum demand when there is no data on unsatisfied demand (the case of limited supply).

The procedures that appear suitable and realistic are (1) using the Gompertz Curve and (2) using the Schmeiser-Deutsch Distribution. We will discuss the Gompertz Curve first.

A. USING THE GOMPERTZ CURVE TO ESTIMATE THE MAXIMUM DEMAND

In the terminology of economics, the demand c.d.f. can be considered as a "growth curve" since it has a "market saturate" as an upper asymptote. The Gompertz Curve is the most generally used growth curve. Its formula

$$Y = ka^{b^X}, \quad (1)$$

where X is the time series and Y is the sales amount, depends on the values of the parameters k , a and b (or their logarithms), it may take on any one of a variety of shapes [Ref. 5].

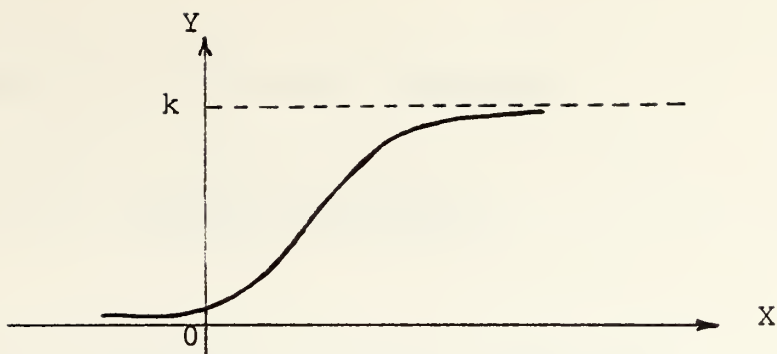


Figure 1. The Gompertz Curve with $0 < a < 1$, $0 < b < 1$

We are interested in using the Gompertz Curve with $0 < a < 1$ and $0 < b < 1$ to approximate the c.d.f. of demand. This form of the Gompertz Curve is known to be asymptotic to k . When we fit the Gompertz Curve to the observed cumulative frequency of the data, k should theoretically be 1.0. In practice, however, it may depart slightly from 1.0. To compensate for this, we use the following form to approximate the c.d.f.:

$$\frac{Y}{k} = a^{b^X}$$

The following description is taken from [Ref. 5]:

The parameters k , a and b are obtained as follows:

- 1) Be certain that the number of observations is divisible by 3, i.e., there are $3N$ periods of base data.
- 2) Take the common logarithms of each Y .
- 3) Add the first N $\log Y$'s to obtain $\Sigma_1 \log Y$; then the second N data points to obtain $\Sigma_2 \log Y$; and the last N points to obtain $\Sigma_3 \log Y$.
- 4) Set the smallest observation to be unity.

5) Substitute in the following formulas:

$$b^N = \frac{\sum_3 \log Y - \sum_2 \log Y}{\sum_2 \log Y - \sum_1 \log Y} , \quad (2)$$

$$\log a = (\sum_2 \log Y - \sum_1 \log Y) \cdot \frac{(b-1)}{(b^N-1)^2} , \quad (3)$$

and

$$\log k = \frac{1}{N} \cdot (\sum_1 \log Y - (\frac{b^N-1}{b-1}) \cdot \log a) . \quad (4)$$

With values of k , a and b , one can compute Y for any value of X using the formula of the Gompertz Curve. Also, the formula can be derived from the Gompertz Curve to generate X from the Gompertz Curve as:

$$X = \frac{\log(\frac{\log Y - \log k}{\log a})}{\log b} \quad (5)$$

In this chapter, we wish to estimate the maximum demand. If the demand distribution has an infinite upper tail, we can not hope to estimate the 100th percentile of demand. We will be content, therefore, with the 99th percentile of demand. This is found by setting $(Y/k) = .99$ or $Y = .99k$ in equation (5), i.e.,

$$X_{\max} \text{ will be } = \frac{\log(\frac{-.0044}{\log a})}{\log b} , \quad (6)$$

which may be considered to be the estimator of the maximum demand if the smallest observed X is unity. (If not, the maximum demand should be adjusted to be X plus the smallest X minus one.)

As an example assume that the demands for newspapers have been 10, 6, 9, 7, 5, 13, 11, 7, 8, 8. Using the Gompertz Curve to estimate the maximum demand, we can proceed as follows:

Let X be the demand for newspapers, and Y be the cumulative relative frequency of X .

TABLE I
DEMAND AND ITS CUMULATIVE RELATIVE FREQUENCY

<u>X</u>	<u>Y</u>
5	0.1
6	0.2
7	0.4
8	0.6
9	0.7
10	0.8
11	0.9
12	0.9
13	1.0

Since Y is a growth curve, it can be approximated by the Gompertz Curve, $Y = ka^b X^b$. To determine the appropriate values of k , a and b using the method described above:

1) From these data, there are 9 distinct values between the smallest observation (5) and the largest observation (13), and 9 is divisible by 3. (Here, $N = 9/3 = 3$.)

2) In the third column of Table II, Y's are converted to log Y's.

3) Values of $\Sigma_1 \log Y$, $\Sigma_2 \log Y$ and $\Sigma_3 \log Y$ are shown in the fourth column of Table II.

4) Since the smallest observed X is not 1.0, the maximum demand should be adjusted by adding the difference between the smallest observed X and 1.0. This is because the Gompertz Curve assumes the smallest observation is unity.

5) From equation (2);

$$\begin{aligned} b^N &= ((-.0915) - (-.4737)) / ((-.4737) - (-2.0969)) \\ &= 0.2355 . \end{aligned}$$

Therefore,

$$b = 0.6175 .$$

From equation (3);

$$\begin{aligned} \log a &= (-.4737 - (-2.0969)) (.6175 - 1) / (.2355 - 1) \\ &= -1.0622 . \end{aligned}$$

Therefore,

$$a = 0.0867 .$$

From equation (4);

$$\begin{aligned} \log k &= \frac{1}{3} (-2.0969 - (.2355 - 1) (-1.0622) / (.675 - 1)) \\ &= 0.0087 . \end{aligned}$$

Therefore,

$$k = 1.0203 .$$

I.e., the Gompertz Curve is $Y = (1.0203)(0.0867)^{(0.6175)^X}$.

Using the 99th percentile, the maximum demand is

$$X_{\max} = 11.3964 + (5-1) = 15.3964$$

$$\approx 15.$$

TABLE II

COMPUTATIONS IN FINDING A GOMPERTZ CURVE

<u>X</u>	<u>Y</u>	<u>Log Y</u>	<u>Σ Log Y</u>
5	0.1	-1.0000	} $\Sigma_1 \log Y = -2.0969$
6	0.2	-0.6990	
7	0.4	-0.3979	
8	0.6	-0.2218	} $\Sigma_2 \log Y = -0.4737$
9	0.7	-0.1549	
10	0.8	-0.0969	
11	0.9	-0.0458	} $\Sigma_3 \log Y = -0.0915$
12	0.9	-0.0458	
13	1.0	0	

A program for the TI-59 to determine the maximum demand when the smallest observed X is unity, and to determine the parameters (k, a and b), is given in Appendix B (Program B1).

Experimentation with simulated data seems to indicate that the Gompertz Curve procedure works well when supply is unlimited, but not nearly so well when supply is limited.

B. USING THE SCHMEISER-DEUTSCH DISTRIBUTION TO ESTIMATE THE MAXIMUM DEMAND

K. Pearson pointed out that a theoretical distribution must have at least four free parameters, in order to fit the four characteristics (location, dispersion, skewness, and kurtosis) adequately [Ref. 6]. Kottas and Lau recommends the four-parameter Schmeiser-Deutsch Distribution as most suitable [Ref. 6]. The details of the Schmeiser-Deutsch Distribution are given in Appendix A.

The Schmeiser-Deutsch Distribution has a finite upper tail. Furthermore, if X is a random variable having the Schmeiser-Deutsch Distribution, there exists a closed-form expression for the maximum value of X . This expression is $a+b(1-d)^c$, where a , b , c and d are the four free parameters.

Using the data in the previous example, we will employ the Schmeiser-Deutsch Distribution to estimate the maximum demand. We first estimate the parameters using the algorithm in Appendix A:

1) The mode $m = (7+8)/2 = 7.5$.

Set $a = 7.5$.

2) Set $d = (Y(7)+Y(8))/2 = 0.5$.

3) Select the two desired quantiles which satisfy equation (9), namely,

$$p_1 = 0.2 , \quad x_1 = 6$$

and

$$p_2 = 0.9 , \quad x_2 = 11$$

Substitute in equation (10), yielding $c = 2.9453$.

4) Substitute in equation (11), yielding $b = 52.0120$.

Then, the maximum demand is

$$\begin{aligned} X_{\max} &= a + b(1-d)^c \\ &= 14.2529 \approx 14. \end{aligned}$$

This compares very well with the result obtained by using the Gompertz Curve which was $X_{\max} = 15.3964$. For these data, there is no substantial difference between these two alternative methods of estimating $X_{\max}$.

A program for the TI-59 to determine the maximum demand and the four parameters of the Schmeiser-Deutsch Distribution is given in Appendix B (Program B2).

C. ESTIMATING MAXIMUM DEMAND WHEN THERE IS NO DATA ON UNSATISFIED DEMAND

In the previous work it was implicitly assumed that the value of demand for each period was obtainable even if demand is greater than supply. This rarely happens in the real world unless backorders are taken. Speaking of the "newsboy problem", the newsboy can collect the data in the range of

demand from zero to ordering quantity (Q) only. When the newspapers are sold out, he will not know how many more were in demand for that day. If he is recording the sales, his observed data on that day will be $X = Q$ even if the demand is actually $X \geq Q$.

The Schmeiser-Deutsch Distribution can be easily used to investigate the case when data for demands greater than the quantity Q are not available. This is because its parameters can be estimated using only the mode and two selected distinct percentiles.

As an example, suppose that the newsboy orders only 10 newspapers for each day. His sales data for a 10 day period are 10, 6, 9, 7, 5, 10, 10, 7, 8, 8. (Instead of demand, which was 10, 6, 9, 7, 5, 13, 11, 7, 8, 8.) The sales data and their cumulative relative frequency will be:

TABLE III

SALES DATA AND THEIR CUMULATIVE RELATIVE FREQUENCY

<u>X</u>	<u>Y(X)</u>
5	0.1
6	0.2
7	0.4
8	0.6
9	0.7
10	1.0

Since $\Pr(X = 10)$ is actually $\Pr(X \geq 10)$, we will ignore this data point.

Following the algorithm in Appendix A for the rest of the data points:

1) Set $a = 7.5$.

2) Set $d = 0.5$.

3) Select $p_1 = 0.6$, $x_1 = 8$, $p_2 = 0.7$, $x_2 = 9$ which satisfy equation (9).

Substitute in equation (10), yielding $c = 1.5850$.

4) Substitute in equation (11), yielding $b = 19.2279$.

From this, the maximum demand is $X_{\max} = 13.9093$ which is close to $X_{\max}$ when the newsboy has unlimited supply.

With this $X_{\max}$, one can determine Q^* . Procedures to do this are discussed in the next chapter.

III. ESTIMATION OF OPTIMAL ORDER QUANTITY

In the previous chapter it was shown how to estimate the maximum demand ($X_{\max}$). In this chapter, the estimate of the optimal order quantity (Q^*) for the case of uncertainty and also for the case of risk will be discussed.

A. OPTIMAL ORDER QUANTITY UNDER UNCERTAINTY

When data in hand is not sufficient to estimate the possible future values of demand or when the distribution of demand is not known, the problem of deciding on Q falls into the class called decisions under uncertainty.

It has been shown that the optimal decisions under uncertainty (Q^*) are [Ref. 1]:

$$Q^* = \left(\frac{c_o}{c_s + c_o} \right) \cdot X_{\max} , \quad \text{for a continuous demand ,}$$

and

$$Q^* < \left(\frac{c_o}{c_s + c_o} \right) (X_{\max} + 1) < Q^* + 1 ,$$

for a discrete demand, where c_s is the unit cost of surplus and c_o is the unit cost of outage.

Since one consequence of each decision is another observation of demand, one begins under uncertainty and uses the uncertainty optimal order quantity until demand data is adequate to estimate the distribution of demand and change to the optimum order quantity under risk.

B. OPTIMAL ORDER QUANTITY UNDER RISK

When one says "under risk", one means that the probability distribution of demand is known or, at least, some estimate of the probability distribution of demand is available. The latter case is possible using the Schmeiser-Deutsch Distribution suggested in Appendix A.

The optimal order quantity under risk is the value of Q^* such that

$$F(Q^*) = \frac{c_o}{c_s + c_o}, \quad \text{for a continuous demand distribution,}$$

and

$$F(Q^*-1) < \frac{c_o}{c_s + c_o} < F(Q^*),$$

for a discrete demand distribution.

It should be noted here that all one really needs to estimate the probability distribution of demand is a value for the $[c_o/(c_o+c_s)]^{\text{th}}$ quantile. The inverse c.d.f. of the Schmeiser-Deutsch Distribution can easily take care of this problem.

As an example, suppose that the newsboy purchased the newspapers at the cost of 10¢ each and he can sell them for 20¢ each, with leftovers having a salvage value of 5¢ each.

Let p be the ratio $[c_o/(c_o+c_s)]$. Therefore, $p = 2/3$.

Consider the data from the previous example at the end of Chapter II. These data are adequate for a decision under

risk. The p^{th} quantile can be estimated by the inverse c.d.f. of the Schmeiser-Deutsch distribution,

$$F^{-1}(p) = \begin{cases} a-b(d-p)^c & \text{if } p \leq d \\ a+b(p-d)^c & \text{if } p > d, \end{cases}$$

where the four parameters were found to be $a = 7.5$,
 $b = 19.2279$, $c = 1.5850$ and $d = 0.5$.

Therefore, $F^{-1}(p) = 8.623$, which is the value of Q^* for the case of continuous demand.

For the case of discrete demand, the optimal order quantity under risk is the value of Q^* such that $F(Q^*-1) < p < F(Q^*)$, and then $Q^* = 9$ in this example.

In the next chapter we will give conclusions and applications from this work.

IV. CONCLUSIONS AND APPLICATIONS

One of the important requirements needed to obtain the optimal order quantity in newsboy problems is the nature of the demand distribution. This thesis suggested some procedures which can be used to estimate the demand distribution even if data on unsatisfied demand are not available. In the case where it is hard to tell what common distribution the demand should belong to, the Schmeiser-Deutsch Distribution was felt to be useful.

It is fitting at this point to explain how the procedures from this and other research on the newsboy problems would be used.

Suppose we are starting a sequence of single-period inventory ordering decisions but have no information on the distribution of demand.

One way of employing these results would involve three phases:

1) For the decisions at time period 1 through some period m , the newsboy would probably rely upon a guess at the value of the maximum demand ($X_{\max}$). Considering the unit cost of surplus (c_s) and the unit cost of outage (c_o), he has two alternatives:

- (a) In case of $c_s \ll c_o$, use $Q^* = X_{\max}$.
- (b) Otherwise, use the uncertainty decision rule.

$$Q^* = [c_o / (c_s + c_o)] \cdot X_{\max} .$$

2) For time periods $(m+1)$ through some later period n , he would use $Q^* = [c_o / (c_o + c_s)] \cdot X_{\max}$ with the value of $X_{\max}$ obtained from the Gompertz Curve or the Schmeiser-Deutsch Distribution, using data from the first m periods. Clearly, he may wish to revise this estimate as more data become available.

3) After time period n , he can switch from treating the decision as a decision under uncertainty to a decision under risk, using the inverse c.d.f. of the Schmeiser-Deutsch Distribution to estimate the value of $[c_o / (c_o + c_s)]^{\text{th}}$ quantile, which is known to be Q^* for continuous demand.

It should be noted here that because of the data needed, m must provide at least three periods where he did not sell out, and from the work of Yong-u Sok, n should provide at least five periods where he did not sell out [Ref. 4].

From the work outlined above we have a reasonable means of starting a sequence of newsboy decisions when there is no information about the demand distribution and lost sales are unknown. It is hoped that the procedures suggested in this paper will be useful to those interested in single-period inventory problems and their solutions.

APPENDIX A

THE SCHMEISER-DEUTSCH DISTRIBUTION

In this appendix, the Schmeiser-Deutsch Distribution will be briefly reviewed. For a further study [Ref. 6] and [Ref. 7] are suggested.

Due to the lack of convenient methods of generating random values from distributions having more arbitrary shapes, the Schmeiser-Deutsch Distribution was developed and found suitable for generating random values from many common statistical distributions.

The Schmeiser-Deutsch Distribution can take on a wide variety of shapes, which makes it useful for many applications. Distributions ranging from Bernoulli trials through U-shaped distributions to the uniform distribution, as well as heavier tailed distributions, are obtainable. Also, shapes ranging from symmetry to maximum skewness, as measured by the third moment, are possible. The Schmeiser-Deutsch Distribution can have values for the first four moments so as to match any of the well-known distributions, and straightforward parameter determination techniques exist.

A most favorable characteristic of the Schmeiser-Deutsch Distributions is the existence of simple closed-form expressions for both the c.d.f. and the inverse c.d.f., being respectively,

$$F(x) = p = \begin{cases} d - [(a-x)/b]^{1/c} & \text{if } (a-bd^c) \leq x \leq a \\ d + [(x-a)/b]^{1/c} & \text{if } a \leq x \leq a+b(1-d)^c \end{cases} \quad (7)$$

and

$$x = F^{-1}(p) = \begin{cases} a - b(d-p)^c & \text{if } p \leq d \\ a + b(p-d)^c & \text{if } p > d \end{cases} \quad (8)$$

where a , b , c and d are the four parameters of the distribution, and p is a percentile [Ref. 7].

"The location and the spread of the distribution are determined by a and b , respectively. The shape of the distribution, often measured in terms of skewness and tail-weight (or peakness), is determined by $-(d-p)^c$ in case of $p \leq d$, or $(p-d)^c$ in case of $p > d$. This factor is a transformation from the rectangular shape of the uniform distribution to the shape of interest. The $d = 0.5$ yields a symmetric distribution. If $c > 1$, then skew is to the right for $d < 0.5$ and to the left for $d > 0.5$. (If $c < 1$, the direction of skew is reversed.)" [Ref. 7]

"The distribution possesses two characteristics which may adversely affect its usefulness for specific modeling purposes. The first characteristic is that the value of the density function evaluated at the mode, $f(x = a)$, assumes

only three values: zero for $0 \leq c < 1$, one for $c = 1$, and infinity for $c > 1$. The second characteristic is the truncated tails of the distribution." [Ref. 7]

"Despite the truncated tails, the distribution's versatility in assuming a wide variety of shapes makes it a reasonable model for a wide range of processes. This is due to the ability to assume the essential features of a process, which often makes the truncation insignificant." [Ref. 7]

Parameter Determination [Ref. 7]

The mode and quantiles are used in estimating parameters for the Schmeiser-Deutsch Distribution. The Schmeiser-Deutsch Distribution is useful in two common situations. The first is when full information is not available, but such quantities as the "most likely value" and the "minimum" and "maximum" values can be estimated. The second situation is when data is in histogram form where $F(m)$ is the percentile of the mode m . The two desired quantiles, $p_1 = F(x_1)$ and $p_2 = F(x_2)$, must satisfy either of the following two expressions:

$$(|a-x_1| < |x_2-a| \quad \text{and} \quad |d-p_1| < |p_2-d|)$$

(9)

$$(|a-x_1| < |x_2-a| \quad \text{and} \quad |d-p_1| < |p_2-d|)$$

Although computations to determine the four parameters can be easily handled by computer, much attention has been given in this thesis to develop "manual procedures", meaning manual computational capability. The TI-59 handheld calculator

(characterized by low price and surprising capability and compactness) can be considered as a "manual" tool, and a program for the TI-59 to determine the four parameters is given in Appendix B.

An algorithm for estimating the parameters is as follows:

[Ref. 7]

- 1) Set $a = m$
- 2) Set $d = F(m)$
- 3) Set

$$c = \ln \left| \frac{a-x_1}{a-x_2} \right| / \ln \left| \frac{d-p_1}{d-p_2} \right| \quad (10)$$

$$4) \text{ Set } b = |a-x_1| / |d-p_1|^c \quad (11)$$

5) Due to the existence of the inverse c.d.f. in closed-form, the value of X corresponding to a percentile p of the distribution may be found by inserting p into equation (8).

Furthermore, the range of X can be calculated easily by substituting $p = 0$ and $p = 1$ at Step 5), or obtained directly from equation (7) since $X_{\min} = a - bd^c$ and $X_{\max} = a + b(1-d)^c$. At this point, one should determine whether or not the obtained distribution is an adequate model by plotting the p.d.f. or c.d.f. [Ref. 7].

APPENDIX B

TI-59 Programs

This appendix presents two TI-59 programs to determine the maximum demand and the parameters. The first program is using the Gompertz Curve with the restriction that the smallest X is unity. The second program is using the Schmeiser-Deutsch Distribution with the two desired quantiles satisfying equation (9).

Program B₁

1. Program Description

The program will estimate the maximum demand using the Gompertz Curve, and will also determine the parameters (k, a and b) in $Y = ka^{b^X}$.

2. User Instruction

<u>Step</u>	<u>Procedure</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
1	Enter n	n	A	n
2	Enter $Y_1, \dots, Y_{3n}$	Y_1	R/S	Y_1
		.	.	.
		.	.	.
		.	.	.
		Y_n	R/S	$\sum_1 \log Y$
		Y_{n+1}	R/S	Y_{n+1}
		.	.	.
		.	.	.
		.	.	.
		Y_{2n}	R/S	$\sum_2 \log Y$

<u>Step</u>	<u>Procedure</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
		Y_{2n+1}	$\boxed{R/S}$	Y_{2n+1}
		$\vdots$	$\vdots$	$\vdots$
		Y_{3n}	$\boxed{R/S}$	$\sum_3 \log Y$
3	Compute $X_{\max}$		$\boxed{B}$	$X_{\max}$
4	Review parameters (optional)			
	k		$\boxed{C}$	k
	a		$\boxed{D}$	a
	b		$\boxed{E}$	b

The maximum demand shown on the display is for the case when the smallest observed X is unity. Otherwise, the maximum demand should be adjusted by adding $X_{\min}^{-1}$, where $X_{\min}$ is the smallest observed X .

000	76	LBL	050	02	2	100	53	(
001	11	A	051	43	RCL	101	53	(
002	47	CM3	052	58	58	102	43	RCL
003	42	STD	053	32	X1T	103	03	03
004	57	57	054	43	RCL	104	75	-
005	42	STD	055	56	56	105	43	RCL
006	58	58	056	67	EQ	106	02	02
007	42	STD	057	17	B'	107	54	)
008	59	59	058	01	1	108	55	+
009	35	1/X	059	94	+/-	109	53	(
010	42	STD	060	44	SUM	110	43	RCL
011	04	04	061	58	58	111	02	02
012	01	1	062	43	RCL	112	75	-
013	42	STD	063	55	55	113	43	RCL
014	56	56	064	61	GTO	114	01	01
015	43	RCL	065	18	C'	115	54	)
016	57	57	066	76	LBL	116	54	)
017	91	R/S	067	17	B'	117	42	STD
018	42	STD	068	43	RCL	118	05	05
019	55	55	069	02	02	119	45	YX
020	28	LDG	070	76	LBL	120	43	RCL
021	44	SUM	071	19	D'	121	04	04
022	01	01	072	91	R/S	122	95	=
023	43	RCL	073	42	STD	123	42	STD
024	57	57	074	55	55	124	06	06
025	32	X1T	075	28	LDG	125	28	LDG
026	43	RCL	076	44	SUM	126	95	=
027	56	56	077	03	03	127	42	STD
028	67	EQ	078	43	RCL	128	07	07
029	16	A'	079	59	59	129	43	RCL
030	01	1	080	32	X1T	130	06	06
031	94	+/-	081	43	RCL	131	75	-
032	44	SUM	082	56	56	132	01	1
033	57	57	083	67	EQ	133	95	=
034	43	RCL	084	10	E'	134	42	STD
035	55	55	085	01	1	135	06	06
036	61	GTO	086	94	+/-	136	43	RCL
037	00	00	087	44	SUM	137	05	05
038	17	17	088	59	59	138	75	-
039	76	LBL	089	43	RCL	139	01	1
040	16	A'	090	55	55	140	95	=
041	43	RCL	091	61	GTO	141	42	STD
042	01	01	092	19	D'	142	05	05
043	76	LBL	093	76	LBL	143	53	(
044	18	C'	094	10	E'	144	43	RCL
045	91	R/S	095	43	RCL	145	02	02
046	42	STD	096	03	03	146	75	-
047	55	55	097	91	R/S	147	43	RCL
048	28	LDG	098	76	LBL	148	01	01
049	44	SUM	099	12	B	149	54	)

150	65	X	200	76	LBL
151	43	RCL	201	14	D
152	06	06	202	43	RCL
153	55	+	203	08	08
154	43	RCL	204	27	INV
155	05	05	205	23	LN _X
156	33	X ²	206	91	R/S
157	95	=	207	76	LBL
158	42	STD	208	15	E
159	08	08	209	43	RCL
160	65	X	210	07	07
161	43	RCL	211	27	INV
162	05	05	212	23	LN _X
163	55	+	213	91	R/S
164	43	RCL	214	00	0
165	06	06			
166	95	=			
167	94	+/-			
168	85	+			
169	43	RCL			
170	01	01			
171	95	=			
172	65	X			
173	43	RCL			
174	04	04			
175	95	=			
176	42	STD			
177	09	09			
178	00	0			
179	93	.			
180	09	9			
181	09	9			
182	28	LOG			
183	55	+			
184	43	RCL			
185	08	08			
186	95	=			
187	28	LOG			
188	55	+			
189	43	RCL			
190	07	07			
191	95	=			
192	91	R/S			
193	76	LBL			
194	13	C			
195	43	RCL			
196	09	09			
197	27	INV			
198	23	LN _X			
199	91	R/S			

Program B2

1. Program Description

Estimate the maximum demand using the Schmeiser-Deutsch Distribution. Also determine the parameters (a, b, c and d) in

$$F(x) = \begin{cases} d - \left[\frac{a-x}{b}\right]^{1/c} & \text{if } a-bd^c \leq x \leq a \\ d + \left[\frac{x-a}{b}\right]^{1/c} & \text{if } a \leq x \leq a+b(1-d)^c \end{cases}$$

2. User Instruction

<u>Step</u>	<u>Procedure</u>	<u>Enter</u>	<u>Press</u>	<u>Display</u>
1	Enter mode m	m	R/S	m
2	Enter F(m)	F(m)	R/S	F(m)
3	Enter p ₁	p ₁	R/S	p ₁
4	Enter x ₁	x ₁	R/S	x ₁
5	Enter p ₂	p ₂	R/S	p ₂
6	Enter x ₂	x ₂	R/S	x ₂
7	Compute X _{max}		E	X _{max}
8	Review parameters (optional)			
	a		A	a
	b		B	b
	c		C	c
	d		D	d

000	42	STD	050	53	(	100	02	2
001	01	01	051	43	RCL	101	85	+
002	91	R/S	052	04	04	102	43	RCL
003	42	STD	053	75	-	103	01	01
004	04	04	054	43	RCL	104	95	=
005	91	R/S	055	07	07	105	91	R/S
006	42	STD	056	54	)	106	76	LBL
007	05	05	057	54	)	107	11	A
008	91	R/S	058	50	IXI	108	43	RCL
009	42	STD	059	23	LNK	109	01	01
010	06	06	060	54	)	110	91	R/S
011	91	R/S	061	42	STD	111	76	LBL
012	42	STD	062	03	03	112	12	B
013	07	07	063	53	(	113	43	RCL
014	91	R/S	064	53	(	114	02	02
015	42	STD	065	43	RCL	115	91	R/S
016	08	08	066	01	01	116	76	LBL
017	91	R/S	067	75	-	117	13	C
018	76	LBL	068	43	RCL	118	43	RCL
019	15	E	069	06	06	119	03	03
020	53	(	070	54	)	120	91	R/S
021	53	(	071	50	IXI	121	76	LBL
022	53	(	072	55	+	122	14	D
023	43	RCL	073	53	(	123	43	RCL
024	01	01	074	53	(	124	04	04
025	75	-	075	43	RCL	125	91	R/S
026	43	RCL	076	04	04	126	00	0
027	06	06	077	75	-	127	00	0
028	54	)	078	43	RCL	128	00	0
029	55	+	079	05	05	129	00	0
030	53	(	080	54	)	130	00	0
031	43	RCL	081	50	IXI	131	00	0
032	01	01	082	45	YX	132	00	0
033	75	-	083	43	RCL	133	00	0
034	43	RCL	084	03	03	134	00	0
035	08	08	085	54	)	135	00	0
036	54	)	086	54	)	136	00	0
037	54	)	087	42	STD	137	00	0
038	50	IXI	088	02	02	138	00	0
039	23	LNK	089	53	(	139	00	0
040	55	+	090	01	1	140	00	0
041	53	(	091	75	-	141	00	0
042	53	(	092	43	RCL	142	00	0
043	43	RCL	093	04	04	143	00	0
044	04	04	094	54	)	144	00	0
045	75	-	095	45	YX	145	00	0
046	43	RCL	096	43	RCL	146	00	0
047	05	05	097	03	03	147	00	0
048	54	)	098	65	X	148	00	0
049	55	+	099	43	RCL	149	00	0

LIST OF REFERENCES

1. Masuda, Junichi, The Single-Period Inventory Model: Origins, Solutions, Variations, and Applications, M.S. Thesis, NPS, September 1977.
2. Lindsay, Glenn F., The Newsboy Problem, NPS, June 1979.
3. Sagdic, Kadir, A Decision Procedure for the Transition from Uncertainty to Risk in a Single-Period Inventory Problem, M.S. Thesis, NPS, September 1978.
4. Sok, Yong-u, A Study of Alternative Quantile Estimation Methods in Newsboy-type Problems, M.S. Thesis, NPS, March 1980.
5. Ullmann, John E., Quantitative Methods in Management, McGraw-Hill Book Company, 1976.
6. Kottas, John F., and Lau, Hon-Shiang, "The Use of Versatile Distribution Families in Some Stochastic Inventory Calculations," Journal of Operational Research Society, Vol. 31, 393-403, May 1980.
7. Schmeiser, Bruce W., and Deutsch, Stuart J., "A Versatile Four Parameter Family of Probability Distributions Suitable for Simulation," AIIE Transactions, Vol. 9, No. 2, 176-182, June 1977.

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